

NOTATION

A, area, m^2 ; a, thickness of the crystallizer, m; a_p , acceleration, m/sec^2 ; F, force, N; f, friction coefficient; g, gravitational acceleration, m/sec^2 ; h, height, m; Δl , free height of the melt layer, m; n, frequency of rotation, sec^{-1} ; R, radius, m; T, temperature, K; T' , temperature of the contact, K; v, velocity, m/sec ; x, coordinate; α , coefficient of heat transfer, $W/(m^2 \cdot K)$; β , slope angle of the casting system, deg; δ , thickness of the melt strip, m; ϵ , rate of cooling, $^{\circ}K/sec$; θ , wetting contact angle, deg; λ , thermal conductivity, $W/(m^2 \cdot K)$; κ , thermal diffusivity, m^2/sec ; ρ , density, kg/m^3 ; i , coefficient of surface tension, N/m; τ , time, sec. Subscripts: a, air; g, gravitation; l, liquid; i, inertia; c, contact; cap, capillary; cr, crystallizer; s, strip; mr, microroughness; st, surface tension; fr, friction; C, centrifugal; p, particle.

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EFFECTIVE THERMAL CONDUCTIVITY OF A STRUCTURED POWDER

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The effective conductivity in a granular bed may be substantially dependent on the transport at granule contacts.

Interest attaches to heat and mass transfer in granular layers because these are widely used in engineering [1-4]; it is difficult to examine theoretically the transport in such media particularly because there are numerous particles, which may pack in various ways, and because there are simultaneous mechanisms differing in nature, whose contributions to the total flux in general are not additive. For example, there may be major components due to conduction in the particles and in the gaps between them, convection in the pores, and phenomena in the contact zones involving surrounding gas and liquid lenses.

A single model cannot incorporate all the phenomena affecting transport here; a more constructive approach involves examining the various mechanisms separately.

The framework conductivity in a granular bed has been examined in experiments on the effective thermal conductivity for a lightly pressed layer of nickel spheres less than a micron in diameter. The pressing has been placed in various media. To insure that the specimen maintained its shape, it was sintered at $750^{\circ}C$ and 10^2 - 10^3 atm. The particles adhered but did not fuse. The pressing increased the contact areas and increased the framework conductivity, while reducing the importance of conduction in the pores. Small parti-

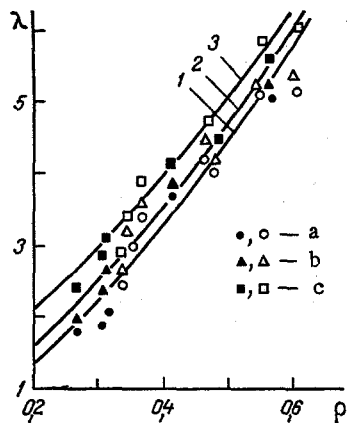


Fig. 1

Fig. 1. Effective thermal conductivity λ of a lightly pressed bed of nickel particles, diameter $2a \approx 0.5 \mu\text{m}$ (filled points) or $0.9 \mu\text{m}$ (open points) as a function of bulk particle content ρ ($b \approx 1.8 a$). The points are from experiment, while the lines are by calculation from (A.1); a and 1 specimen in air, b and 2 in alcohol, c and 3 in water.

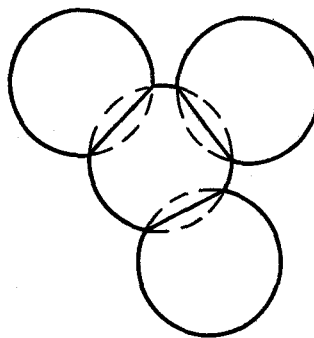


Fig. 2

Fig. 2. Sphere contact model, where the dashed lines are the geometrical continuations of the spheres in the adjacent ones.

cles were used to make the specimen statistically representative and to reduce the effects from fluctuations in porosity and structure.

Figure 1 shows the results; estimates from conforming methods [1-6] exceeded the values by factors of 5-10. Results analogous to ours have been obtained in measurements [7] under similar conditions.

The [1-6] methods have been tested in numerous experiments with large particles and well-sintered ceramics, so the [7] results and ours indicate that a very fine powder with pronounced boundaries between particles shows important effects from the contact zones, which have elevated thermal resistance. The increased resistance at the contacts may be due to microroughness, oxide or adsorption layers, or scattering and diffraction for the heat carriers at the contacts (phonons and conduction electrons), or to other factors.

Finely divided powders and various ceramics are common engineering materials; our measurements and the [7] data however show that the conductivity is not always described by traditional methods. It is therefore necessary to devise a method of calculating the effective conductivity that incorporates the contacts.

In [1-6] and in [8-10] there are surveys of researches on effective conductivity determination for powders, but the approaches there are based on models and essentially heuristic assumptions. They are suitable for engineering calculations, but the large number of intuitive assumptions does not allow one to generalize from them. The heuristic and semi-empirical approaches are inadequate, and they should be used only when no suitable method can be selected. For example, in [7], the measurements were processed by means of Odelevskii's formula [11], which is justified only for matrix mixtures having moderate dispersed-phase concentrations [12] and in principle is incorrect for a framework system. The accidental agreement between measurements and what is certainly a theory inapplicable under these conditions prevents one from performing a physical analysis of the situation and creates spurious views on heat transfer in the material.

A logical description can be obtained only from a correct theory that does not employ unsound assumptions; ensemble averaging [13, 14] meets these conditions in describing the thermophysical parameters of a flow composed of isolated spherical particles. In [15, 16], that theory was extended to framework materials, but subsequent tests showed that the contact model used there was too simplified and mostly failed to describe the actual situation.

That deficiency in [15, 16] is corrected here. The formalism in that theory has been described in detail in [15-17], so here we give only the main physical concepts and the results.

Physical Model. The specimen is considered as a system of identical randomly dispersed spheres forming a homogeneous and isotropic framework in a continuous matrix. We assume that the inclusions are in contact as intersecting spheres (Fig. 2); we neglect any deformation outside the contact regions.

One needs a detailed description of the contact processes in a strict formulation, but no theory exists for the thermophysical parameters of these contact spots. Also, in most practical cases one does not know the geometrical or physical features of the particle surfaces or contact areas.

The task is substantially simplified if the thickness ℓ of the region in which the conductivity is altered at the contact is much less than the particle radius a , which we subsequently assume; then one can neglect the contact-zone thickness and consider that zone as infinitely thin and having surface conductivity κ .

The surface conductivity has been defined for example in [18] for electrophysical processes; there is a complete analogy between heat and mass transport in a phenomenological description, so we consider the surface thermal conductivity, which is formally defined as for the electrical conductivity in [18]. Without entering into details, we note that κ can have either sign: positive if the surface layer has conductivity higher than that of the particle material and vice versa. Also, κ does not have the dimensions of the ordinary thermal conductivity.

An important point is that the contact layer thickness is small in the physical sense, not the mathematical one, so the layer has a finite (nonzero) thermal resistance, and the temperatures at the two surfaces are different. In general, the boundary condition for the temperature at the contact surface of one sphere with another is

$$T_+ - T_- = \alpha \beta n \nabla T_-; T_{\pm} = \lim_{r \rightarrow a_{\pm}} T. \quad (1)$$

Here β is a parameter that in general is not zero; (1) follows from dimensional arguments and also from the obvious circumstance that the temperatures on the two sides of the contact are the same if there is no heat flux through the surface ($n \nabla T_- = 0$).

It is not our purpose to determine κ and β explicitly; they are considered as empirical parameters, but β can be estimated as follows. We represent the contact surface as a homogeneous layer, thickness ℓ and thermal conductivity λ_s . To a first approximation, λ_s is given by $\lambda_s \sim \lambda_1 + \kappa/\ell$. We consider the balance between the fluxes on the two sides to get

$$\beta \sim \frac{\ell}{a} \frac{\lambda_1}{\lambda_1 + \kappa/\ell}. \quad (2)$$

We note that (2) is an estimate, perhaps crude; it can be used only for qualitative analysis.

Macroscopic Thermal Conduction Equation. The general theory [13] gives equations describing the heat transfer in the continuum approximation derived by averaging equations applicable to the pores and particles together with the boundary conditions at the surface, where one averages over all permissible configurations. See [16, 17] for the derivation for this medium. We omit details and write the final result for the case where the Fourier number constructed in the two phases on scales a can be neglected:

$$c \frac{\partial \tau}{\partial t} = -\nabla \mathbf{q}; \mathbf{q} = -\lambda \nabla \tau; c = c_0(1 - \rho) + c_1 \rho, \quad (3)$$

where λ is defined by the formal relations

$$\begin{aligned} \lambda \nabla \tau(\mathbf{R}) = & \lambda_0 \left[\nabla \tau(\mathbf{R}) - \frac{\rho}{v} \int_{r=a} x \tau^+(\mathbf{R} + \mathbf{r}) d\mathbf{r} \right] + \lambda_1 \frac{\rho}{v} \int_{r \leq a} \nabla \tau^*(\mathbf{R} + \mathbf{r}) d\mathbf{r} + \\ & + \frac{\kappa}{2} \rho_1 \rho \int_{r=a} \text{div}_s \text{grad}_s [\tau^*(\mathbf{R} + \mathbf{r}) + \tau^+(\mathbf{R} + \mathbf{r})] d\mathbf{r}; \quad v = \frac{4}{3} \pi a^3. \end{aligned} \quad (4)$$

Here τ^+ and τ^* are the temperatures outside and inside an arbitrarily selected trial particle subject to the condition that there is averaging over the positions of the other particles, while x is the cartesian coordinate r parallel to the mean flux q .

Trial-Particle Treatment. We formulate the determination of τ^+ and τ^* in accordance with [13, 14] by averaging the local heat-transfer equations and the boundary conditions at the particles over the permissible configurations subject to the condition that the position of one of the particles is fixed. We again omit details, which are to be found in [17], and write the final result:

$$\begin{aligned} \Delta\tau^*(r) &= 0; \quad r \leq a; \quad \nabla\hat{q}(r) = 0, \quad r \geq a; \\ \tau^*(r) + a\rho_1\beta n\nabla\tau^*(r) &= \tau(r) + \hat{\tau}(r), \quad nq^*(r) = n[q(r) + \hat{q}(r)] + \rho_1 \frac{\kappa}{2a} \operatorname{div}_s \operatorname{grad}_s [\tau^*(r) + \tau^+(r)], \quad r = a; \\ \hat{\tau} &\rightarrow 0, \quad r \rightarrow \infty; \quad \tau^* < \infty; \quad \hat{\tau} \equiv \tau^+ - \tau; \quad q^* = -\lambda_1 \nabla\tau^*; \quad \hat{q} = -\lambda'(r) \nabla\hat{\tau}; \\ \lambda'(r) &= \lambda_0 + \frac{\rho'(r)}{\rho} (\lambda - \lambda_0); \quad \rho_1 \equiv \rho'(a); \\ \tau(r), \tau^*(r), \tau^+(r) &\equiv \tau(R+r), \tau^*(R+r), \tau^+(R+r). \end{aligned} \quad (5)$$

Then (5) formally coincides with the treatment for a single particle placed in an infinite medium whose properties are in some way controlled by the form of the conditional dispersed-phase concentration $\rho'(r)$, which are thus dependent on the distance to the particle center. The surface has a thin layer with surface conductivity $\kappa\rho_1$ and resistance parameter $\beta\rho_1$ (see [18] for a detailed derivation for the boundary condition on the fluxes when the surface conductivity is incorporated). As $\rho'(r)$ near the particle is not equal to ρ , which is entirely due to the particle being incompletely permeable, the specified sphere drives away the others. Then $\rho'(r) \rightarrow \rho$ for $r \rightarrow \infty$, which is a direct consequence of the correlation dying away. Then κ and β appear in (5) with weight ρ_1 , since ρ_1 is equal to the fraction of the surface area occupied by contact spots.

By definition [13, 14], $\rho'(r)$ is

$$\rho'(r) = \int_{|r'-r| \leq a} \varphi(r') dr', \quad (6)$$

where $\varphi(r')$ is the probability density of finding the center of at least one particle in r' if the center of the trial sphere is at the origin. The form of $\varphi(r')$ is determined by the particular packing and is taken as given here. One uses a suitably chosen φ in (6) to derive τ^* and τ^+ from (5) and (6), which are then used in (4) to calculate λ .

One often gets the situation where one can specify the coordination number ξ , which is equal to the mean number of contacts per unit particle surface, and the mean distance b ($b < 2a$) between the centers of contacting spheres. A simple form for φ corresponding to such packing is

$$\begin{aligned} \varphi(r') &= \xi\delta(r' - b) + \begin{cases} 0, & r' < 2a, \\ \frac{\rho}{v}, & r' > 2a, \end{cases} \\ v &= \frac{4}{3} \pi a^3, \end{aligned} \quad (7)$$

where $\delta(x)$ is a delta function.

We substitute (7) into (6) and then into (5) to get equations that can be solved only numerically; to simplify calculating the smooth function $\rho'(r)$, it can be approximated as a step one. The appendix gives the explicit form of $\rho'(r)$ corresponding to (6) and (7) together with a possible approximation, in which the space around the trial particle is split up into several concentric layers (equal to the number of steps), in which ρ' is taken as constant. We incorporate the continuity conditions for the temperature $\hat{\tau}$ and flux \hat{q} at the boundary of each layer. In a three-step approximation, one determines λ from eight linear equations and four nonlinear ones, which are given in the appendix. This system is readily solved by computer.

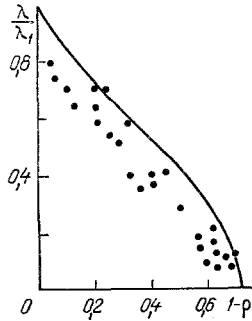


Fig. 3

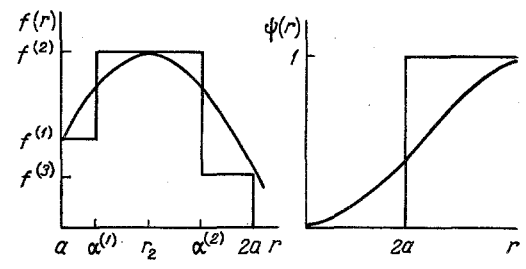


Fig. 4

Fig. 3. Comparison of (A.1) calculations for $\kappa, \beta = 0$ with experiment [5] (Figs. 4-6 in that book); ξ and b have been chosen in accordance with the [5] recommendations.

Fig. 4. Choice of step approximation for $\rho'(r)$ used in appendix.

The κ and β appearing in (5) and thus in (4) for λ are considered here as empirically specified constants; the condition $l \ll a$ in (2) implies that good surface conductivity means $(\kappa/a \geq \lambda_1)$ $\beta \sim l/a \ll 1$, and this quantity can be neglected in the calculations. If on the other hand the contact layer has elevated resistance ($\beta \geq 1$), (2) implies that $|\kappa/a| \approx \sigma_1 l/a$ and one can neglect κ/a . Therefore, there is only one empirically defined parameter for practical calculations: κ or β .

Figure 1 shows measurements, which have been described with $\kappa = 0$; β was chosen to give the best agreement with the theory when the specimen was in air, which gave $\beta \approx 9.8$. These β were used to describe the experiments with water and alcohol. Figure 1 also shows the calculations, which agree satisfactorily with the measurements.

An independent check was provided by comparison with [5] measurements on a highly sintered ceramic, where there were no physically prominent boundaries between particles, so $\kappa, \beta = 0$. (A.1) was also used. Figure 3 shows the characteristic result, which also agrees well with the theory. There is a certain systematic excess because the three-step $\rho'(r)$ approximation is crude.

This method gives a fairly cumbersome system, which becomes even more complicated when one raises the $\rho'(r)$ approximation accuracy, which is clearly a deficiency, although the complexities are purely computational. In essence, the approach enables one to incorporate many factors that complicate the transport here, e.g., (A.1) enables one to calculate the trace-component diffusion coefficient for a polycrystalline material on the basis that the diffusion in the contact zones is easier than in the crystallites, with $\beta \approx 0$, while κ is chosen by fitting theory to a simple measurement or is estimated from independent considerations. In principle, it is also quite simple to incorporate sorption-desorption conditions at the particles, if the diffusion and sorption equations are averaged as for example in [19].

As the contact zones influence the conductivity substantially, one needs research on the physical properties of the surface and contact areas under various conditions; such research can provide a basis for controlling the properties, in addition to the purely scientific interest.

APPENDIX

The $\rho'(r)$ given by (6) and (7) is

$$\rho'(r) = \xi \pi f(r) + \rho \psi(r), \quad \pi = 3,14 \dots,$$

$$f(r) = \left(b - r - \frac{b^2 - a^2 - r^2}{2r} \right)^2 + 2 \left[a^2 - \left(\frac{b^2 - a^2 - r^2}{2r} \right)^2 \right],$$

$$r \in \Omega = \left[\frac{2b^2 - a^2 - a \sqrt{2b^2 + a^2}}{2b}, \frac{2b^2 - a^2 + a \sqrt{2b^2 + a^2}}{2b} \right],$$

$$f(r) = 0, r \notin \Omega; \psi(r) = \frac{27 - 56\eta + 30\eta^2 - \eta^4}{16\eta},$$

$$1 < \eta \equiv \frac{r}{a} < 3; \psi(r) = 0, \eta < 1; \psi(r) = 1, \eta > 3.$$

The smooth functions f and ψ are approximated by steps (Fig. 4):

$$f(r) = \begin{cases} 0, & r < a \\ f^{(1)}, & a < r < \alpha^{(1)} & \alpha^{(1)} = \frac{a + \sqrt{b^2 - a^2}}{2}, \\ f^{(2)}, & \alpha^{(1)} < r < \alpha^{(2)} & \alpha^{(2)} = \frac{b + \sqrt{b^2 - a^2}}{2}, \\ f^{(3)}, & \alpha^{(2)} < r < \alpha^{(3)} & \alpha^{(3)} = 2a, \\ 0, & r > \alpha^{(3)} & f^{(j)} = f(r_j), j = 1, 2, 3, \\ & & r_1 = a; r_2 = \sqrt{b^2 - a^2}, r_3 = 2a; \end{cases}$$

$$\psi(r) = \begin{cases} 0, & a < r < 2a, \\ 1, & r < 2a. \end{cases}$$

These representations can be used with (4) and (5) to calculate λ from a system in which A, B, C, D, E, F, H, and G are considered as unknowns along with $\lambda^{(j)}$ and λ , while the other quantities are taken as parameters. A detailed derivation has been given for a system analogous to that below but with $\kappa, \beta = 0$ in [17]:

$$\begin{aligned} A(1 + \beta\rho_1) - B - Ca^{-3} &= 1, \quad A\lambda_1 - (B - 2Ca^{-3})\lambda^{(1)} + \\ + \rho_1 \frac{\kappa}{2a}(A + B + Ca^{-3} + 1) &= \lambda, \quad B + C\alpha^{(1)-3} - D - E\alpha^{(1)-3} = 0, \\ \lambda^{(1)}(B - 2C\alpha^{(1)-3}) - \lambda^{(2)}(D - 2E\alpha^{(1)-3}) &= 0, \\ D + E\alpha^{(2)-3} - F - G\alpha^{(2)-3} &= 0, \\ \lambda^{(2)}(D - 2E\alpha^{(2)-3}) - \lambda^{(3)}(F - 2G\alpha^{(2)-3}) &= 0, \quad F + G\alpha^{(3)-3} - H\alpha^{(3)-3} = 0, \\ \lambda^{(3)}(F - 2G\alpha^{(3)-3}) + 2H\alpha^{(3)-3}\lambda &= 0, \\ \lambda^{(j)} &= \lambda_0 + (\lambda - \lambda_0) \frac{\pi \xi f^{(j)}}{\rho}, \quad j = 1, 2, 3, \\ \lambda &= \lambda_0 [1 - \rho(1 + B + Ca^{-3})] + \rho\lambda_1 A + \rho\rho_1 \frac{\kappa}{2a}(1 + A + B + Ca^{-3}). \end{aligned} \tag{A.1}$$

NOTATION

a , particle radius; b , mean distance between centers of contacting spheres; c, c_1 , and c_0 , effective specific heat, specific heat of particle material, and specific heat of matrix; l , contact zone thickness, where the conductivity differs from that of the particle material; n , unit vector normal to particle surface; q , mean flux; \hat{q} , perturbation introduced by trial particle into q pattern; R , radius vector in laboratory coordinate system; r , radius vector from particle center; T , true temperature; β , parameter characterizing contact-layer thermal resistance; λ and λ' , effective thermal conductivity and conductivity near trial sphere; λ_1 and λ_0 , thermal conductivities of particle material and medium in pores; ρ and ρ' , volume concentrations of dispersed phase on average and near trial particle; τ and $\hat{\tau}$, mean temperature and perturbation introduced by trial sphere into τ pattern; $\varphi(r)$, binary sphere distribution. The operators div_s and grad_s coincide with the usual divergence and gradient operators but for the fixed value $r \equiv a$.

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OPTIMIZATION OF THERMOCOUPLE INSTALLATION FOR STUDY OF INTENSE
TRANSIENT THERMAL ACTIONS ON MATERIALS

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A method is proposed for determination of the optimum variant of thermocouple installation to reduce measurement error for the case of individual electrode welding.

In studying thermal processes in various devices and equipment the temperature and thermal fluxes in the most heavily loaded details often cannot be measured directly. For example, such a situation is typical of heat transport analysis on the surfaces of cutting and abrasive instruments, and in portions of casting machines in contact with the solidifying metal. The difficulties in experimental determination of such quantities usually involve rapid destruction of sensors by thermal or mechanical action or insufficient measurement accuracy [1]. In such situations it becomes desirable to perform indirect measurements and process them by converse problem methods [2].

One way to improve effectiveness of such studies is planning, by which we understand selection of the basic experimental factors [3]: the quantities to be measured directly, the number and location of primary transducers, special features of loading. For tempera-

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